

The Pythagorean Theorem and Physics

The Pythagorean Theorem defines separation relations that must prevail if the universe is to exist. Some properties of Space resulting therefrom are explored.

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1 Foreword

It is known by direct observation that stars in their galaxies have been and are accumulating to the outer fringes of their galaxies at a constant acceleration within the galaxy. The current speed reached by the stars in the Milky Way is 200 km/sec. Likewise the galaxies themselves are collectively receding from their point of origin in the universe at a constant acceleration and also repelling one-another with a constant acceleration of the same value as that of the stars within a galaxy. Scientists have attributed the phenomena to forces produced by so-called dark matter and energy.

This account is wrong.

The actual explanation can be found by exploring Clifford Algebra and its role in quantizing the gravitational field. In the manuscript, "The Pythagorem Theorem and Physics," it is shown that there exists a real force that does not interact with Newtonian Gravity but does exert a negative gravitational force that is responsible for the observed expansion phenomena. Preparatory to this explanation, the document shows that gravity has the same structure as electromagnetism and may be similarly quantized.

In addition, the 43 seconds of arc advance per century of the Perihelion of Mercury, not included in the 5,500 seconds of arc predicted by Newtonian gravitational theory, is accounted for. This work occupies the major portion of the manuscript. Essential contributions come from Clifford Algebra and the validity of Fermat's Last Theorem which was only proven as late as the mid 1990s. The anomalous behavior of the galaxies is discussed in detail in the latter portion of the document.

A summary of key concepts from the manuscript are presented below.

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2 General Comments and Facts

2.1 General Comments

The Orbital Period of Mercury is 224.7 days (0.615 yrs). Its Rotatation Period is 243 days. The tilt of its axis is 3° . It has no satellite. Mercury precesses 43 seconds of arc per century farther than the 5,500 seconds calculated from Newtonian Theory. If the gravitational constant G is changed to $G(1+6v^2/c^2)$ where v is the velocity of the Perihelion, it yields the unexplained advance of 43 seconds of arc per century. Sources of v^2/c^2 that add up to $6v^2/c^2$ are listed in Section 2.4 of the text. Most of the manuscript is devoted to deriving these corrections.

In the process, new results are obtained. For example, the Schwarzschild metric appears when curvature is included in the space-time metric. Fermat's Last Theorem, proven in the mid 1990s, plays a critical role by enabling one to use the Pythagorean Theorem as a basic property of space. Clifford algebra and the associated multi-vectors are constraints on nature. See the text for details.

2.2 Facts

In the text, Newton's 3rd law that action and reaction are equal and opposite is verified to an accuracy of v^2/c^2 , all that is necessary in the present work. The expansion can be readily extended to a higher order.

The sun is about 26,000 light years from the center of the galaxy. The Milky Way formed five billion years after the Big Bang.

Three solar masses are required to form a mini black hole. There are roughly 2.5 million solar masses in the center of the Milky Way.

2.3 Kepler's Laws (1609)

1. Planets orbit the sun in ellipses with the sun at one focus.
2. The line joining the sun and a planet sweeps out equal areas in equal times.
3. The square of the period of revolution is proportional to the cube of the semimajor axis of the ellipse, that is $P^2 \propto R^3$

Derivation of the 3rd Law

For a particle in a circle, that is, neglecting elliptical motion since $m_s = 300,000m_e$

$$GMm/R^2 = mv^2/R$$

$$v = \sqrt{GM/R}$$

Since distance = velocity \times time, the time to complete a period is:

$$2\pi R = (GM/R) P$$

$$4\pi^2 R^2 = GMP^2/R$$

$$4\pi^2 R^3 = GMP^2$$

$$P^2 = (4\pi^2/GM) R^3$$

Apply this result to the earth's rotation around the sun, where $R = 1.5 \times 10^8 km$.

From the earth's orbital speed $v = 30 km/sec$, we deduce the sun's mass:

$$M_{sun} = M_s = 2 \times 10^{23} \text{grams} (\approx 350,000 \text{ earth masses})$$

Mass of a galaxy = $v^2 R/G$ within a radius R from its center.

R = distance of sun from its center = 27,000 light years, $v = 200 km/sec$

From the experimental rotation curve, $R = 27,000$ light years, thus mass within sun's orbit is

$$m =$$

$$(200 km/sec)^2 \times (10^3 cm/km^2) \times (27,000 \text{ lt yr}) \times \frac{9.5 \times 10^{27} cm/lt \text{ yr}}{6.673 \times 10^{-8} cm^3/gm/sec^2}$$

The mass of the sun = $2 \times 10^{33} gm$, thus the mass of the galaxy is 7.4×10^{10} solar masses = $7.56 \times 10^{44} gm$ or $\approx 10^{11}$ solar masses. Sampling the density of galaxies, it is found that there are approximately 10^{11} galaxies in the Universe.

3 Overview

In the following, we list a few ideas employed in the text and the results therefrom that lead to the quantization of gravity and also explain why the Universe of galaxies has been, and is, expanding at a constant acceleration and why the stars in their galaxies have been and are accumulating in the outer fringes of the galaxies at a constant acceleration.

We begin by accounting for the 43 seconds of arc advance per century of the perihelion of Mercury not included in the observed 5600 seconds of arc per century, by changing the Newtonian G to $G(1 + 5v^2/c^2)$ and a relativity correction of $G(1 + v^2/c^2)$. We show that the larger correction follows from classical physics plus a contribution from space-time algebra, defined herein.

The contributions of the various sources needed to obtain an effective G of $G(1 + 5v^2/c^2)$ are summarized in Section 2.4. To obtain the force between two moving charges, we use the Biot-Savart Law, Section 8.1, and Faraday's Law, Section 8.2.

Another important idea is that two algebras are necessary to describe the interaction between two moving entities. One algebra describes the motion of the entities and a second algebra describes the property of the particular entity being transported. The algebras are independent of one another and, therefore, commute. This formalism is described in Section 8.3. We also distinguish between two types of forces: a one on one, or direct force, and the others implemented by a field, an electromagnetic field in the case of electromagnetism. Gravity interactions will have a gravitational field with the same structure as electromagnetism. It is at this stage that we derive Eq. 8.16 where a "source" particle, designated by subscript s , is acting on a "test" particle, designated by subscript t . However, the equation also contains a contribution from a trivector with a bivector charge. This is not a vector and must be converted to a vector. We call the procedure for doing this contraction and it is described in Section 8.5. It is also shown at this stage how property charges q_s , q_t , whatever they may represent, enter the formalism.

Gravity is described in Section 8.8. All of the material discussed in the text is important. For details, we refer the reader to the text. The problem of properly describing gravity is inherently complicated.

Chapter 14 is a partial summary and describes again how a new gravitational force appears. It exerts a negative force between gravitational masses and is responsible for the accelerated expansion of the Universe and the accelerated accumulation of stars at the outer fringes of the galaxies. It is a real force. The idea of introducing dark matter and energy to explain the accelerated expansion phenomena is dismissed as ad hoc and unnecessary to account for the observed expansion.

Fermat's Last Theorem plays a critical role in this work. Fermat's Last Theorem states that the equation $z^n = x^n + y^n$ has no integer solution for $n > 2$. Fermat (1601-1665) claimed he had a simple proof but that it was too long to write in the margin of his book. His proof, right or wrong, has never been found. The theorem was finally proven by Andre Wiles in 1994, utilizing a long complicated proof.

Fermat's Theorem is our starting point, namely that distances add quadratically. It is an intrinsic property of space, that is, of space-time where we introduce time as an ordering parameter.

Fermat's Theorem leads to Clifford Algebra, an important formalism for the present work. In Chapter 1, it is shown that the Pythagorean Theorem may be expressed in terms of anti-commuting unit vectors, e_1 and e_2 , to which we later add e_3 for a third dimension, plus an ordering parameter e_0 . This leads to an algebra, which we call space-time algebra,

consisting of five types of multi-vectors:

scalar:	$i S$
vectors:	$\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$
bivectors:	$\mathbf{e}_0\mathbf{e}_1, \mathbf{e}_0\mathbf{e}_2, \mathbf{e}_0\mathbf{e}_3, \mathbf{e}_1\mathbf{e}_2, \mathbf{e}_3\mathbf{e}_1, \mathbf{e}_2\mathbf{e}_3$
trivectors:	$i (\mathbf{e}_0\mathbf{e}_1\mathbf{e}_2, \mathbf{e}_0\mathbf{e}_3\mathbf{e}_1, \mathbf{e}_0\mathbf{e}_2\mathbf{e}_3, \mathbf{e}_1\mathbf{e}_2\mathbf{e}_3)$
quadvector:	$i \mathbf{e}_0\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3$

An $i = \sqrt{-1}$ must be selectively inserted as shown for each group to be a conserved quantity, see Section 6.11

Since the above entities follow uniquely from the Pythagorean Theorem and since the Pythagorean Theorem is a unique property of space-time, we assert that all properties of nature must be described by space-time algebra.

An important fact is that a conserved current has an associated field. For example, a moving charged mass, such as an electron, has an associated electromagnetic field described by Maxwell's equations, (see Eqs. (5.11-5.14) of the text.) This statement follows from the first Bianci identity given by Equation (5.37), text, where the identity is used but not specified by name. There are two Bianci identities:

$$\begin{aligned} \square \bullet \square \bullet \mathbf{F} &= 0 \\ \square \wedge \square \wedge \mathbf{F} &= 0 \end{aligned}$$

where

$$\square = \mathbf{e}_0 \frac{\partial}{c\partial t} + \mathbf{e}_1 \frac{\partial}{\partial x} + \mathbf{e}_2 \frac{\partial}{\partial y} + \mathbf{e}_3 \frac{\partial}{\partial z}$$

The definition of the dot, (inner), product of two multivectors a, b is

$$\mathbf{a} \bullet \mathbf{b} = (\mathbf{ab} + \mathbf{ba}) / 2$$

The wedge, (outer), product is defined as

$$\mathbf{a} \wedge \mathbf{b} = (\mathbf{ab} - \mathbf{ba}) / 2$$

Of the two Bianci identities, we use only the first.

Our immediate goal is to properly describe gravity, a vector quantity. The only viable candidate of the five multivectors for this role is the 4-velocity vector:

$$\mathbf{V}_s = \mathbf{e}_0 c + \mathbf{e}_1 v_{sx} + \mathbf{e}_2 v_{sy} + \mathbf{e}_3 v_{sz}$$

multiplied by "charge" m_s . It exerts an attractive force on another 4-velocity vector

$$\mathbf{V}_t = \mathbf{e}_0 c + \mathbf{e}_1 v_{tx} + \mathbf{e}_2 v_{ty} + \mathbf{e}_3 v_{tz}$$

multiplied by charge m_t .

The product in turn is multiplied by an inverse r^2 factor, the distance between the two masses. The same observation applies to the force between two electrical charges but with opposite sign. The fact that the separation distance r must enter as $1/r^2$ is shown in the text. In describing interactions, we will often omit writing the $1/r^2$ factor, but it is understood to be present.

Another critical observation is that a conserved current implies the existence of an associated field. All multivectors form conserved currents and, therefore, in principle, have fields. The form of the fields that result by virtue of conserved currents follow from the Bianci identities given above where F is any multivector or linear combination of multivectors in space-time algebra.

When dealing with vectors, as opposed to multivectors, either the inner product, \bullet , or the Gibb's dot product, \cdot , may be used. For our purposes, we choose to use the inner product. \square is the space-time gradient operator.

Referring to

$$\square \bullet \square \bullet \mathbf{F} = 0,$$

if

$$\square \bullet \mathbf{F} = \mathbf{j}$$

then

$$\square \bullet \mathbf{j} = 0$$

and current is conserved.

The only viable candidate for F out of the five space-time vectors is the electromagnetic field bivector

$$\mathbf{F} = (\mathbf{e}_0\mathbf{e}_1E_x + \mathbf{e}_0\mathbf{e}_2E_y + \mathbf{e}_0\mathbf{e}_3E_z) / c + \mathbf{e}_1\mathbf{e}_2B_z + \mathbf{e}_3\mathbf{e}_1B_y + \mathbf{e}_2\mathbf{e}_3B_x$$

In conventional language, F is the antisymmetric electromagnetic field tensor. We are talking about gravity, but we use the same terminology as electromagnetism.

Electromagnetism and gravity have the same form. Only the charge differs. For gravity, the charge is mass. For electromagnetism, it is electric charge, where the charge has an effective mass.

The 4-current in the above equation is

$$\mathbf{J} = \mathbf{e}_0\mu_0c^2\rho + \mu_0\mathbf{j} = \mathbf{e}_0\frac{\rho}{\epsilon_0} + \mu_0\mathbf{j}$$

Thus, gravity can be quantized since it has the same structure as electromagnetism.

In the text, Newton's 3rd law that action and reaction are equal and opposite is verified to an accuracy of v^2/c^2 all that is necessary in the present work. The expansion can be readily extended to higher orders.

A few facts: the Milky Way is 90,000 light years in diameter. The sun is about 26,000 light years from the center. According to the Big Bang Theory, its age is 13.73 billion years. The Milky Way formed 5 billion years after the Big Bang. Three solar masses are required to form a mini black hole. There are roughly 2.5 million solar masses forming the black hole at the center of the Milky Way.

4 Structure of Gravity

In this work we present an introduction to space-time algebra and explain its role in quantizing the gravitational field. It is also shown how a real force gives rise to negative gravity and eliminates the need for fictitious dark matter and energy to account for the observation that stars in the Milky Way and other galaxies have been and are accumulating in their outer fringes at a constant, but very small acceleration within the galaxies. Likewise the universe of galaxies are collectively receding from their point of origin at a constant acceleration and are also repelling one-another with a constant acceleration of the same numerical value as that of the stars within the galaxies.

We argue that gravity satisfies equations identical to electromagnetism on a microscopic scale. However, the microscopic currents effectively add to zero in a macroscopic particle of mass so that only the charge, that is, the mass itself remains. One must keep in mind that when we use the phrase Maxwell's Equations when referring to gravity, we mean that gravity has the same structure, that is, the same form as Maxwell's Equations. The elements of mass m_s and m_t are effectively charges. Current is the velocity of the charges.

The acceleration referred to above is very small but has been in effect since the formation of the galaxies. It may be calculated as follows. The Milky Way, for example, was formed 5 billion years after the Big Bang.

The Universe is now 13.73 billion years. Therefore, the accumulation of the stars at the fringes has been going on for 8.73 billion years. Velocity is acceleration times time. By direct measurement, the current speed reached at the fringes of the Milky Way is 200 km/sec. Therefore, acceleration in km/sec is 200 km/sec divided by 8.73 billion years expressed in seconds. The number of seconds in 8.73×10^6 years is $2.876 \times 10^{14} sec$. Thus the acceleration has been and is $a = 7 \times 10^{-13} km/sec^2$.

The present work began with the goal of accounting for the 43 missing seconds of arc per century when Newtonian Gravity is used to calculate the advance of the perihelion of Mercury. These sources of are listed in Section 2.14.

Up to the mid-nineties, the only tools and materials available for describing nature were vectors and scalars. In order to satisfy a conserva-

tion equation, the scalar must be multiplied by the prefix $i = \sqrt{-1}$. With the advent of Clifford Algebra, it is shown that other entities must be added to vectors and scalars. The entities are given by the 16 elements of space-time algebra:

$$\begin{aligned} \mathcal{L} = & iS + v_0\mathbf{e}_0 + v_1\mathbf{e}_1 + v_2\mathbf{e}_2 + v_3\mathbf{e}_3 \\ & + B_{01}^{E_x}\mathbf{e}_0\mathbf{e}_1 + B_{02}^{E_y}\mathbf{e}_0\mathbf{e}_2 + B_{03}^{E_z}\mathbf{e}_0\mathbf{e}_3 + B_{12}^{B_z}\mathbf{e}_1\mathbf{e}_2 + B_{31}^{B_y}\mathbf{e}_3\mathbf{e}_1 + B_{23}^{B_x}\mathbf{e}_2\mathbf{e}_3 \\ & + i[T_{123}\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3 + T_{023}\mathbf{e}_0\mathbf{e}_2\mathbf{e}_3 + T_{031}\mathbf{e}_0\mathbf{e}_3\mathbf{e}_1 + T_{012}\mathbf{e}_0\mathbf{e}_1\mathbf{e}_2] \\ & + i\mathbf{e}_0\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3 S' \end{aligned}$$

that is;

$$\mathcal{L} = \begin{array}{cccccc} i \text{ scalar} & + & \text{vector} & + & \text{bivector} & + & i \text{ trivector} & + & i \text{ quadvector} \\ \text{imaginary} & & \text{real} & & \text{real} & & \text{imaginary} & & \text{imaginary} \end{array}$$

The coefficients of the scalar element, the trivector elements, and the quadvector elements are pure imaginary (must be multiplied by $\sqrt{-1}$). The coefficients of the vector elements and the bivector elements are real. These modifications are necessary in order that each element in the algebra satisfies a continuity equation.

$$\frac{\partial}{\partial x_\mu} \tilde{\mathcal{L}} \mathbf{e}_u^{-1} \mathcal{L} = 0$$

In order that these results describe nature, Fermat's Last Theorem must be unequivocally true. The theorem was only proven as recently as 1994 by Andrew Wiles. With these results one can state that the Pythagorean Theorem describes an intrinsic property of, or constraint on, space itself. We are interested in gravity which is conventionally described by Newton's Law of Gravitation. It states that the masses

m_t, m_s of any size attract one another with a force $F_{ts} = \frac{-Gm_t m_s}{r^2}$. It is shown that several sources of correction are required to obtain $F_{m_t m_s} = \frac{-Gm_t m_s}{r^2} \left(1 + 6\frac{v^2}{c^2}\right)$ which agrees with experimental data (see section 2.4).

It is important to keep in mind that the multivectors only form the basis for some physical property. For example, a vector may be the basis for a force and must be multiplied by a numerical coefficient specifying magnitude. Line two in \mathcal{L} could represent the electromagnetic field tensor. As we shall see later, line three can represent a force.