

The Pythagorean Theorem and Physics

The Pythagorean Theorem defines separation relations that must prevail if the universe is to exist. Some properties of Space resulting therefrom are explored.

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