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NEGATIVE GRAVITATION: SUPPLEMENTARY REMARKS

A trivector, $\mathbf{T}$, may be written:

$$\begin{aligned}
\mathbf{T} &= T^{012}\mathbf{e}_0\mathbf{e}_1\mathbf{e}_2 + T^{031}\mathbf{e}_0\mathbf{e}_3\mathbf{e}_1 + T^{023}\mathbf{e}_0\mathbf{e}_2\mathbf{e}_3 + T^{123}\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3 \\
&= \mathbf{e}_5\left(T^{012}\mathbf{e}_3 + T^{031}\mathbf{e}_2 + T^{023}\mathbf{e}_1 + T^{123}\mathbf{e}_0\right) \\
&= \mathbf{e}_5\left(T^{123}\mathbf{e}_0 + T^{023}\mathbf{e}_1 + T^{031}\mathbf{e}_0 + T^{012}\mathbf{e}_3\right) \\
&= \mathbf{e}_5\left(u_0\mathbf{e}_0 + u_x\mathbf{e}_1 + u_y\mathbf{e}_2 + u_z\mathbf{e}_3\right)
\end{aligned}$$

where $\mathbf{e}_5 = \mathbf{e}_0\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3,\ \ u_0 = T^{123},\ \ u_1 = T^{023},\ \ u_2 = T^{031},\ \ u_3 = T^{031}$

21.1 Direct Product of 2 Space-time Multivectors

$$\begin{aligned}
\mathbf{uv} &= \left(\mathbf{uv} + \mathbf{uv}\right)/2 + \left(\mathbf{uv} - \mathbf{vu}\right)/2 \\
\mathbf{uv} &= \left(u_0\mathbf{e}_0 + u_1\mathbf{e}_1 + u_2\mathbf{e}_2 + u_3\mathbf{e}_3\right)\left(v_0\mathbf{e}_0 + v_1\mathbf{e}_1 + v_2\mathbf{e}_2 + v_3\mathbf{e}_3\right) \\
&= +u_0v_0\mathbf{e}_0\mathbf{e}_0 + u_0v_1\mathbf{e}_0\mathbf{e}_1 + u_0v_2\mathbf{e}_0\mathbf{e}_2 + u_0v_3\mathbf{e}_0\mathbf{e}_3 \\
&\quad +u_1v_0\mathbf{e}_1\mathbf{e}_0 + u_1v_1\mathbf{e}_1\mathbf{e}_1 + u_1v_2\mathbf{e}_1\mathbf{e}_2 + u_1v_3\mathbf{e}_1\mathbf{e}_3 \\
&\quad +u_2v_0\mathbf{e}_2\mathbf{e}_0 + u_2v_1\mathbf{e}_2\mathbf{e}_1 + u_2v_2\mathbf{e}_2\mathbf{e}_2 + u_2v_3\mathbf{e}_2\mathbf{e}_3 \\
&\quad +u_3v_0\mathbf{e}_3\mathbf{e}_0 + u_3v_1\mathbf{e}_3\mathbf{e}_1 + u_3v_2\mathbf{e}_3\mathbf{e}_2 + u_3v_3\mathbf{e}_3\mathbf{e}_3
\end{aligned}$$

$$\begin{aligned}
\mathbf{vu} &= (v_0\mathbf{e}_0 + v_1\mathbf{e}_1 + v_2\mathbf{e}_2 + v_3\mathbf{e}_3)(u_0\mathbf{e}_0 + u_1\mathbf{e}_1 + u_2\mathbf{e}_2 + u_3\mathbf{e}_3) \\
&= +v_0u_0\mathbf{e}_0\mathbf{e}_0 + v_0u_1\mathbf{e}_0\mathbf{e}_1 + v_0u_2\mathbf{e}_0\mathbf{e}_2 + v_0u_3\mathbf{e}_0\mathbf{e}_3 \\
&\quad +v_1u_0\mathbf{e}_1\mathbf{e}_0 + v_1u_1\mathbf{e}_1\mathbf{e}_1 + v_1u_2\mathbf{e}_1\mathbf{e}_2 + v_1u_3\mathbf{e}_1\mathbf{e}_3 \\
&\quad +v_2u_0\mathbf{e}_2\mathbf{e}_0 + v_2u_1\mathbf{e}_2\mathbf{e}_1 + v_2u_2\mathbf{e}_2\mathbf{e}_2 + v_2u_3\mathbf{e}_2\mathbf{e}_3 \\
&\quad +v_3u_0\mathbf{e}_3\mathbf{e}_0 + v_3u_1\mathbf{e}_3\mathbf{e}_1 + v_3u_2\mathbf{e}_3\mathbf{e}_2 + v_3u_3\mathbf{e}_3\mathbf{e}_3
\end{aligned}$$

$$(\mathbf{uv} + \mathbf{vu})/2 = -u_0v_0 + u_1v_1 + u_2v_2 + u_3v_3 = \mathbf{u}\cdot\mathbf{v} = \text{Scalar}$$

Forms a “dot Product,” same as with normal vectors.

$$\begin{aligned}
(\mathbf{uv} - \mathbf{vu})/2 &= (v_0u_1 - v_1u_0)\,\mathbf{e}_0\mathbf{e}_1 + (v_0u_2 - u_2v_0)\,\mathbf{e}_0\mathbf{e}_2 + (v_0u_3 - u_3v_0)\,\mathbf{e}_0\mathbf{e}_3 \\
&\quad +\frac{1}{2}(u_1v_2 - v_1u_2)\,\mathbf{e}_1\mathbf{e}_2 + \frac{1}{2}(u_1v_3 - v_1u_3)\,\mathbf{e}_1\mathbf{e}_3 \\
&\quad +\frac{1}{2}(u_2v_1 - v_2u_1)\,\mathbf{e}_2\mathbf{e}_1 + \frac{1}{2}(u_2v_3 - v_2u_3)\,\mathbf{e}_2\mathbf{e}_3 \\
&\quad +\frac{1}{2}(u_3v_1 - v_3u_1)\,\mathbf{e}_3\mathbf{e}_1 + \frac{1}{2}(u_3v_2 - v_3u_2)\,\mathbf{e}_3\mathbf{e}_2 \\
&= (u_1v_2 - v_1u_2)\,\mathbf{e}_1\mathbf{e}_2 + (u_3v_1 - v_3u_1)\,\mathbf{e}_3\mathbf{e}_1 \\
&\quad + (u_2v_3 - v_2u_3)\,\mathbf{e}_2\mathbf{e}_3 \\
&\quad + (v_0u_1 - v_1u_0)\,\mathbf{e}_0\mathbf{e}_1 + (v_0u_2 - u_2v_0)\,\mathbf{e}_0\mathbf{e}_2 + (v_0u_3 - u_3v_0)\,\mathbf{e}_0\mathbf{e}_3 \\
&= \text{Bivector} = \mathbf{u}\wedge\mathbf{v}
\end{aligned}$$

$$\therefore \quad \mathbf{uv} = \mathbf{u}\cdot\mathbf{v} + \mathbf{u}\wedge\mathbf{v} = (\mathbf{uv} + \mathbf{vu})/2 + (\mathbf{uv} - \mathbf{vu})/2 = \ \mathit{Scalar}{+}\mathit{Bivector} \tag{21.1}$$

21.2 Vector, Trivector and Contraction. Alternate Discussion

A key idea herein is that the force of gravity is produced by a space-time 4-dimensional momentum "charge." Thus the force of attraction between masses M_s and M_t is not simply GM_sM_t but is given by $(P_sP_t + P_tP_s)/2$.

At the time of the Big Bang, two forces appeared:

a) Gravitational attractive force described by a 4-momentum vector "charge" associated with each mass. This will yield conventional Newtonian gravity but with a correction term. The 4-momentum vector charge P_s is given by $P_s = M_sV_s$ where V_s is a 4-velocity, $V_s = (e_0c + v_s)$. The subscript labels one of the quantities, arbitrarily called the source particle. The second particle is labeled with a subscript t, for test particle. Both masses (scalars) must be multiplied by i to be conserved quantities (see Section 6.11). Therefore, their product is negative, corresponding to an attractive force. The negative sign is a consequence of writing $GM_s\dot{M}_t$ with a positive sign.

b) A second interaction, a trivector, is associated with each mass. As a trivector, it must be multiplied by i twice, once to conserve its mass and again simply because it is a trivector. The square of a unit trivector is -1. A general trivector T_s is

$$T_s = T_s^{012}e_0e_1e_2 + T_s^{031}e_0e_3e_1 + T_s^{023}e_0e_2e_3 + T_s^{123}e_1e_2e_3$$

The square of each trivector component is -1. (See Section 8.13, for further details). They are mutually orthogonal.

A general interaction, in this case a force, between two multivectors, 4-vectors V_s and V_t, may be written:

$$V_sV_t = (V_sV_t + V_tV_s)/2 + (V_sV_t - V_tV_s)/2 \tag{21.2}$$

The velocities are written in non-heavy case letters since they refer to property values q_s and q_t. The interaction $(V_sV_t + V_tV_s)/2$ is always a vector when V_s, V_t are 4-vectors. A rule has been given by Greider and Morris, called contraction (see Section 8.3), that guarantees that the second term in Eq. (21.2) will be a multi-vector of like kind and sign so that it matches the first term.

Now write explicitly the scalar product of V_s, V_t. If:

$$\begin{aligned}
V_s &= (ce_0 + v_{sx}e_1 + v_{sy}e_2 + v_{sz}e_3) = (ce_0 + v_s) \\
V_t &= (ce_0 + v_{tx}e_1 + v_{ty}e_2 + v_{tz}e_3) = (ce_0 + v_t) \\
(V_sV_t + V_tV_s)/2 &= (ce_0 + v_s)(ce_0 + v_t) + (ce_0 + v_t)(ce_0 + v_s) \\
&= e_0^2c^2 + v_{sx}v_{tx} + v_{sy}v_{ty} + v_{sz}v_{tz} = \left(-c^2 + v_s \cdot v_t\right) \\
(V_sV_t + V_tV_s)/2 &= -c^2\left(1 - \frac{v_s \cdot v_t}{c^2}\right)
\end{aligned}$$

Now convert V_s, V_t to momenta by multiplying by iM_s and iM_t, respectively. This is required because M_s, M_t,being scalars, each needs a coefficient i. As previously stated, the i guarantees the conservation of M_s and M_t. Designate the result by P_s and P_t, then:

$$\begin{aligned}
(P_sP_t + P_tP_s)/2 &= iM_siM_t\left[-c^2 + (v_s \cdot v_t)/c^2\right] \\
&= (-1)\left(-c^2\right)M_sM_t\left[1 - (v_s \cdot v_t)/2\right] \\
&= \text{Positive force (attractive)}
\end{aligned} \tag{21.3}$$

Thus when P_s, P_t are interacting 4-momentum vectors, the force between them is attractive (standard gravitational force). After being multiplied by iM_s and iM_t, if the first term in Eq. (21.2) is attractive, the second term, *after contraction*, will also be attractive. We evaluate only the first term, since it is algebraically simpler. For the final magnitude of the force, both terms are required, that is $(P_sP_t + P_tP_s)/2$ and $(P_sP_t - P_tP_s)/2$.

To repeat, at the time of the Big Bang two kinds of gravitational forces appeared:

a) The standard attractive vector force between two charges q_s and q_t which in their usual form appear as two masses attracting each other in Newton's law of gravitation. In the present work we regard the mutual force as mediated by a 4-dimensional space-time gravitational momentum "charge." Four dimensional means 3-space and one time component. The vector force may act on mass or energy. The mass equivalent of energy is obtained by dividing the Planck relation $h\nu$ =energy by c.

b) A second term, a trivector is mediated by a 4-dimensional space-time trivector "charge." The trivector has no field and therefore creates no radiation. Masses repel one another rather than attract.

In summary, the trivector interaction has been present since the time of the Big Bang and may manifest itself as a source of parity between stars and galaxies (conjecture).