18
FACTOR OF i IN TABLES.

18.1 On the Use of the Tables in Chapter 8

The factor $i = \sqrt{-1}$, p. 94, has not been included as a multiplier of the coefficient of the scalar, trivector, and quadvector terms in Tables I through IV in Chapter 8. It, as well as the mass or effective mass, may be inserted by the user. Any quantity that designates a "property," that is, a particular value for q_s and q_t, is written in light case letters to distinguish from the carrier algebra.

For example, the "charge" P_s for gravity is given by:

$$\begin{aligned} q_s \rightarrow P_s &= im_s\left(e_0 c + v_x e_1 + v_y e_2 + v_z e_3\right) \\ &= im_s\left(e_0 c + v_s\right) \end{aligned}$$

The gravitational force of charge P_s on charge P_t is given by Term I in Eq. (8.16). That is, by

$$\begin{aligned} \mathbf{F}_{ts} &= \left(\frac{\mathbf{B}_s\mathbf{V}_t - \mathbf{V}_t\mathbf{B}_s}{2}\right)\left(\frac{P_sP_t + P_tP_s}{2}\right) = \frac{(\mathbf{B}_s\mathbf{V}_t - \mathbf{V}_t\mathbf{B}_s)}{2}P_sP_t \\ P_sP_t &= im_s im_t\left(e_0 c + v_s\right)\left(e_0 c + v_t\right) \\ &= (i)^2\, m_s m_t\left(-c^2 + v_s \cdot v_t\right) \\ &= -\,(i)^2\, c^2 m_s m_t\left(1 - v_s \cdot v_t\right) \\ &= c^2 m_s m_t\left(1 - \frac{v_s \cdot v_t}{2}\right) \end{aligned}$$

Thus Term I has the form

$$\mathbf{F}_{ts} = c^2 \left(\frac{\mathbf{B}_s \mathbf{V}_t - \mathbf{V}_t \mathbf{B}_s}{2} \right) m_s m_t \left(1 - \frac{v_s \cdot v_t}{2} \right)$$

This has the necessary positive sign for gravity.

Next consider Term II in Eq. (8.16).

$$\left(\frac{\mathbf{B}_s \mathbf{V}_t + \mathbf{V}_t \mathbf{B}_s}{2} \right) (q_s q_t - q_t q_s) /2 = \left(\frac{\mathbf{B}_s \mathbf{V}_t + \mathbf{V}_t \mathbf{B}_s}{2} \right) \left(\frac{P_s P_t - P_t P_s}{2} \right)$$

q_s and q_t are prefixed by a scalar iq_s and iq_t respectively.

$$\begin{aligned} q_s q_t - q_t q_s &= i m_s i m_t \left(e_0 c + v_s \right) \left(e_0 c + v_t \right) - \left(e_0 c + v_t \right) \left(e_0 c + v_s \right) \\ &= (i)^2 m_s m_t \left[\left(-c^2 + v_s \cdot v_t \right) - \left(-c^2 + v_t \cdot v_s \right) \right] \\ &= (i)^2 m_s m_t [0] = 0 \end{aligned}$$

Therefore, for gravity, Term II in Eq. (8.16) is zero and makes no contribution. No contraction procedure is necessary. This is true for any pair of like multivectors.

This observation is verified in a different way in Chapter 14. See Eq. (14.67).